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## XII.

ON THE LIMITS OF ACCURACY IN MEASUREMENTS  
WITH THE MICROSCOPE.

BY PROFESSOR EDWARD W. MORLEY, OF WESTERN RESERVE COLLEGE.

Presented Oct. 9, 1878.

THE following measurements of rulings on glass, by Mr. Rogers, were made with an objective of two tenths of an inch focus, and a cobweb micrometer. For a description of the ruled plates the reader is referred to page 178 of the present volume of the Proceedings. Light was thrown on the rulings by reflection from clouds: care was taken to have the light as uniform as possible. Such care is necessary in making accurate measurements with a lens of short focus. The screw for fine adjustment was permitted to be moved only through half a revolution during the measurements. The same parts of the micrometer screw were used throughout the measurements of a band. The image of the line ruled on the plate consists of a bright central line, with a darker line on each side; the wires of the micrometer were placed on this central brighter line, and so near its apparent left-hand limit that the bright line included between the dark wire and the dark border of the image of the ruled line was the minimum visible quantity, and was the same for both wires. Care was taken not to look at the index of the micrometer until the coincidence of the wires was finally established; and also to move the wires a considerable quantity before making a second measurement, except in perhaps five cases on the third plate. In two cases the coincidence thus finally established was re-examined after the reading had been taken, on account of divergence from a previous result, and in one of these the coincidence was found to be imperfect. With this exception, the figures given are absolutely the whole of the measurements on the rulings.

Two bands on the first plate, and one each on the second and third plates, were measured twice. The probable difference of two measurements of the same interval was found to be one three hundred and fifty-nine thousandth of an inch; from which the probable error of a single measurement may be presumed to be about two millionths of an inch. It happened that thirty-five of the differences between two measurements of the same space were less than the probable difference as computed by the usual formula, and thirty-five were greater.

The measurements on the third plate were more difficult than the other, partly because the lines were too fine for the easiest work, and partly on account of fatigue. They are, therefore, less satisfactory than the measurements on the other plates. The outer lines of some bands on the second plate were also troublesome, and the results for two or three not so good as for other spaces.

Mr. Rogers made measurements of the same plates, which he prepared for publication without knowing my results, but after the original micrometer readings of my measurements had passed beyond my control. Of his measurements I know nothing at the time of writing the following tabular results. By concert with him, my results are tabulated in the form adopted by him, for ease of comparison. My numbers for the spaces measured increase in the direction of the arrows on the ruled plates, if I have made no mistake, and also my numbers of the bands. The numbers of the plates are those written on them by Mr. Rogers. In the third plate I measured only spaces composed of five of the spaces of one twenty-four hundredth of an inch as ruled; the difficulty of the measurement of so faint lines, as well as the fear of incurring a return of a certain slight difficulty with one of my eyes, from which recovery was not then complete, led me thus to abridge this part of the work. It is to be regretted that this plate was not taken in hand earlier.

The figures in the columns of individual and accumulated errors represent millionths of an inch composed of twenty-four revolutions of the screw of Mr. Rogers's ruling engine. But in the case of the fourth plate they represent hundred-thousandths of a millimetre of a similar standard.

P L A T E I.									
S P A C E.	B A N D I.			B A N D I I.			B A N D I I I.		
	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.
1	-2	-2	-3	-3	-3	-3	+3	+3	8
2	-6	-8	-10	-13	-10	-13	-10	-13	7
3	-13	-21	-13	-26	-27	-27	-34	-34	34
4	-8	-29	-21	-47	-16	-50	50	50	64
5	-16	-45	-26	-73	-14	-64	64	64	95
6	-23	-68	-28	-101	-31	-95	95	95	109
7	-18	-86	-16	-117	-21	-116	116	116	150
8	-27	-113	-24	-141	-29	-145	145	145	182
9	-18	-131	-26	-167	-21	-166	166	166	208
10	-7	-138	-19	-186	-20	-186	186	186	239
11	-12	-150	-14	-200	-12	-194	194	194	240
12	-12	-162	-12	-212	-9	-203	203	203	235
13	-12	-174	-6	-218	-1	-204	204	204	235
14	-2	-176	7	-211	-5	-209	209	209	231
15	+4	-172	+12	-199	+12	-197	197	197	213
16	+15	-157	+22	-187	+23	-174	174	174	213
17	+20	-137	+27	-150	+20	-154	154	154	182
18	+19	-118	+15	-135	+26	-128	128	128	182
19	+21	-97	+20	-115	+28	-100	100	100	182
20	+18	-79	+26	-89	+28	-72	72	72	126
21	+20	-59	+25	-64	+23	-49	49	49	96
22	+20	-39	+21	-43	+20	-29	29	29	69
23	+17	-22	+19	-24	+20	-7	7	7	40
24	+18	-4	+21	-3	+16	+1	1	1	20
25	+3	-1	+3	0	+0	+0	0	0	4

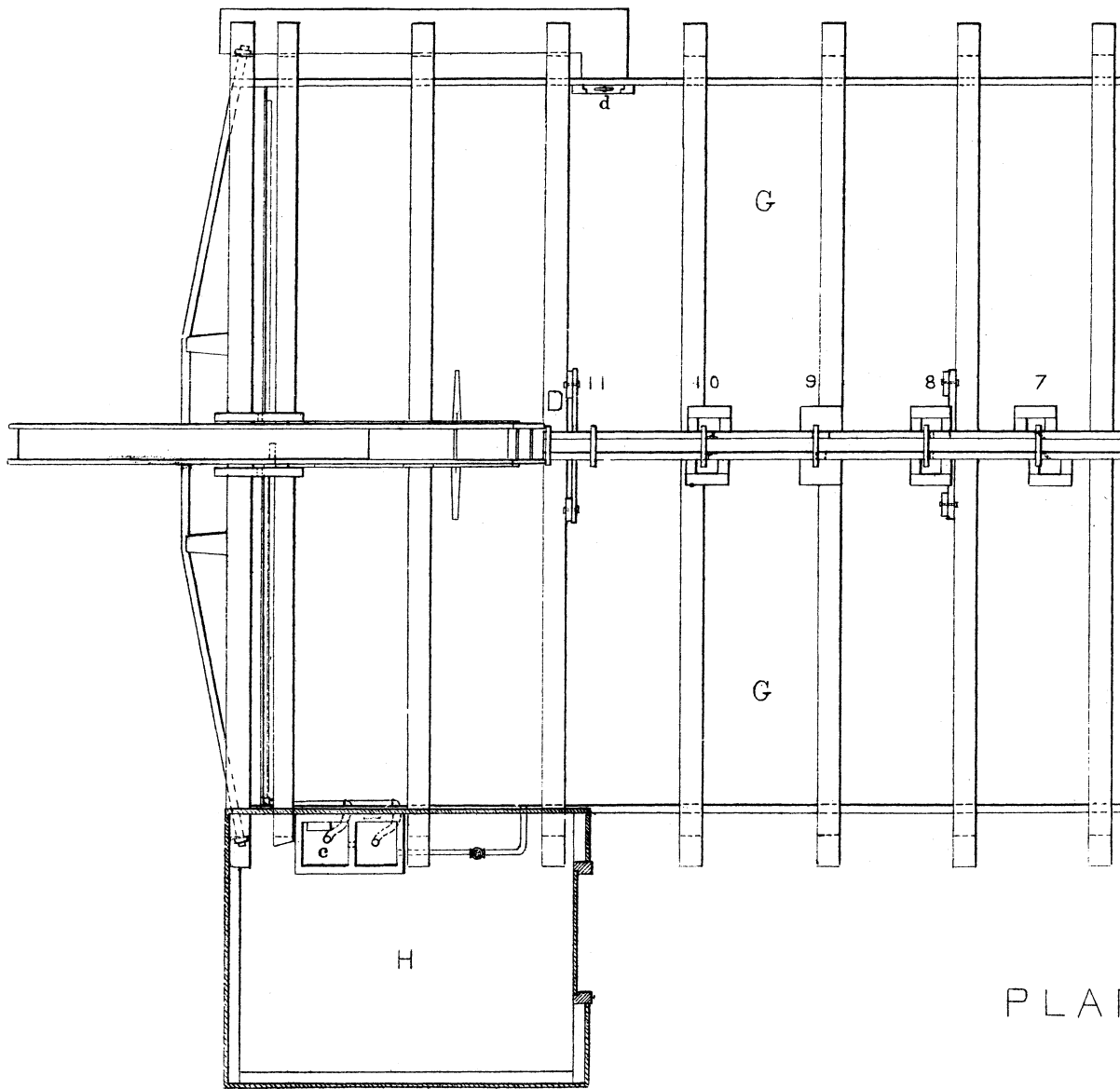
  

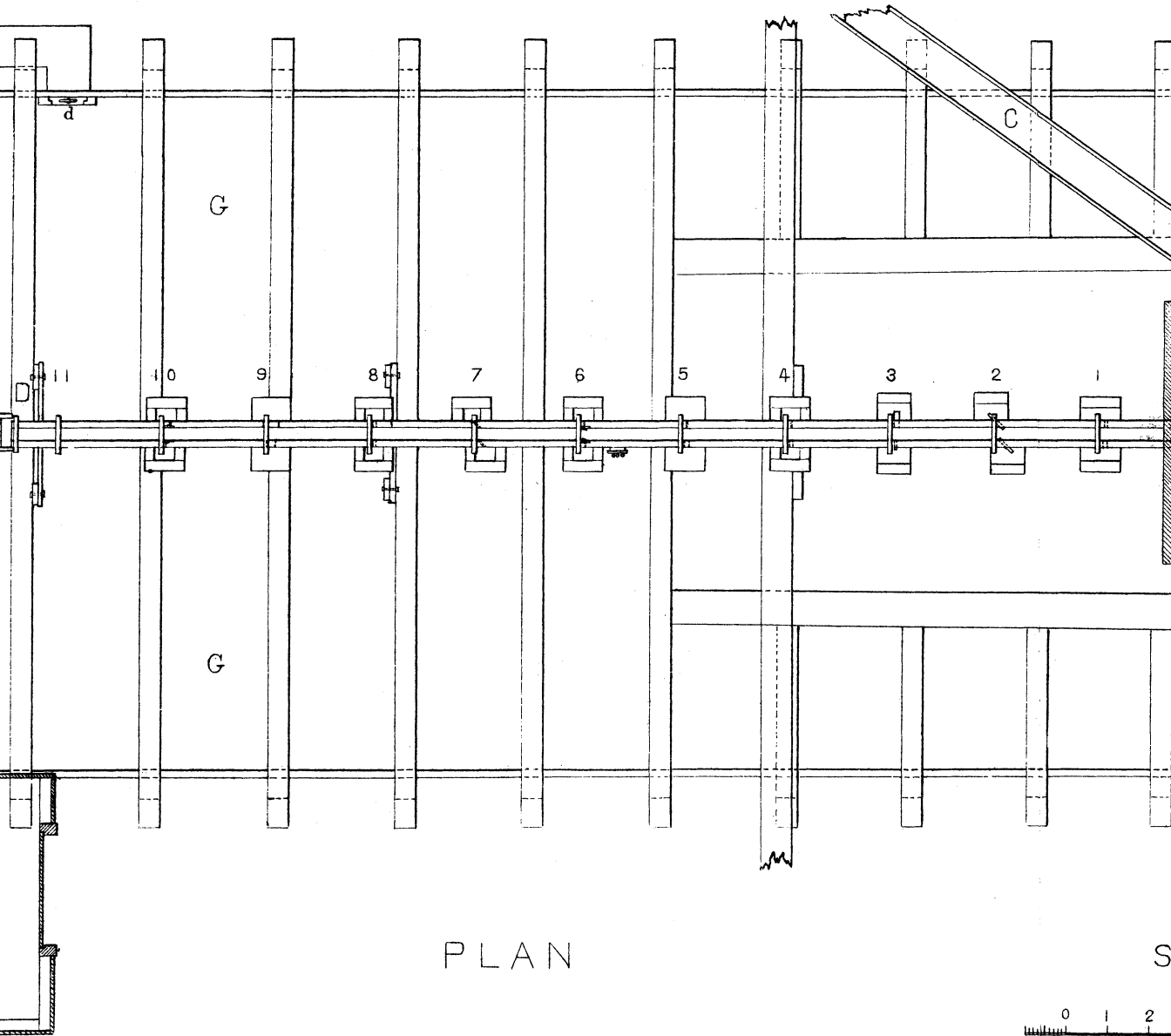
P L A T E I.									
	B A N D I V.			B A N D V.			B A N D V I.		
	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.
	+11	+11	+7	+7	+1	+2	+2	+2	+2
	-19	-8	-14	-7	-18	-16	-14	-18	-16
	-15	-23	-15	-22	-21	-38	-24	-38	-38
	-19	-42	-23	-45	-21	-59	-25	-63	-28
	-27	-69	-18	-63	-33	-92	-33	-96	-36
	-27	-96	-27	-90	-38	-130	-33	-129	-31
	-22	-118	-16	-106	-28	-158	-37	-166	-25
	-33	-151	-30	-136	-20	-178	-22	-188	-19
	-20	-171	-25	-161	-17	-195	-16	-204	-12
	-8	-179	-13	-174	-16	-211	-14	-218	-14
	-3	-182	-6	-180	-7	-204	-8	-210	-4
	-2	-184	+1	-179	+13	-191	+17	-193	+12
	+15	-169	+17	-162	+19	-172	+20	-173	+27
	+21	-148	+22	-140	+30	-142	+29	-144	+32
	+26	-122	+29	-111	+26	-116	+25	-119	+29
	+29	-93	+28	-83	+33	-83	+31	-88	+28
	+32	-61	+29	-54	+28	-55	+29	-59	+25
	+23	-38	+20	-34	+25	-30	+23	-36	+18
	+27	-11	+25	-9	+20	-10	+22	-14	+13
	+12	+1	+5	-4	+16	+6	+11	-3	+14

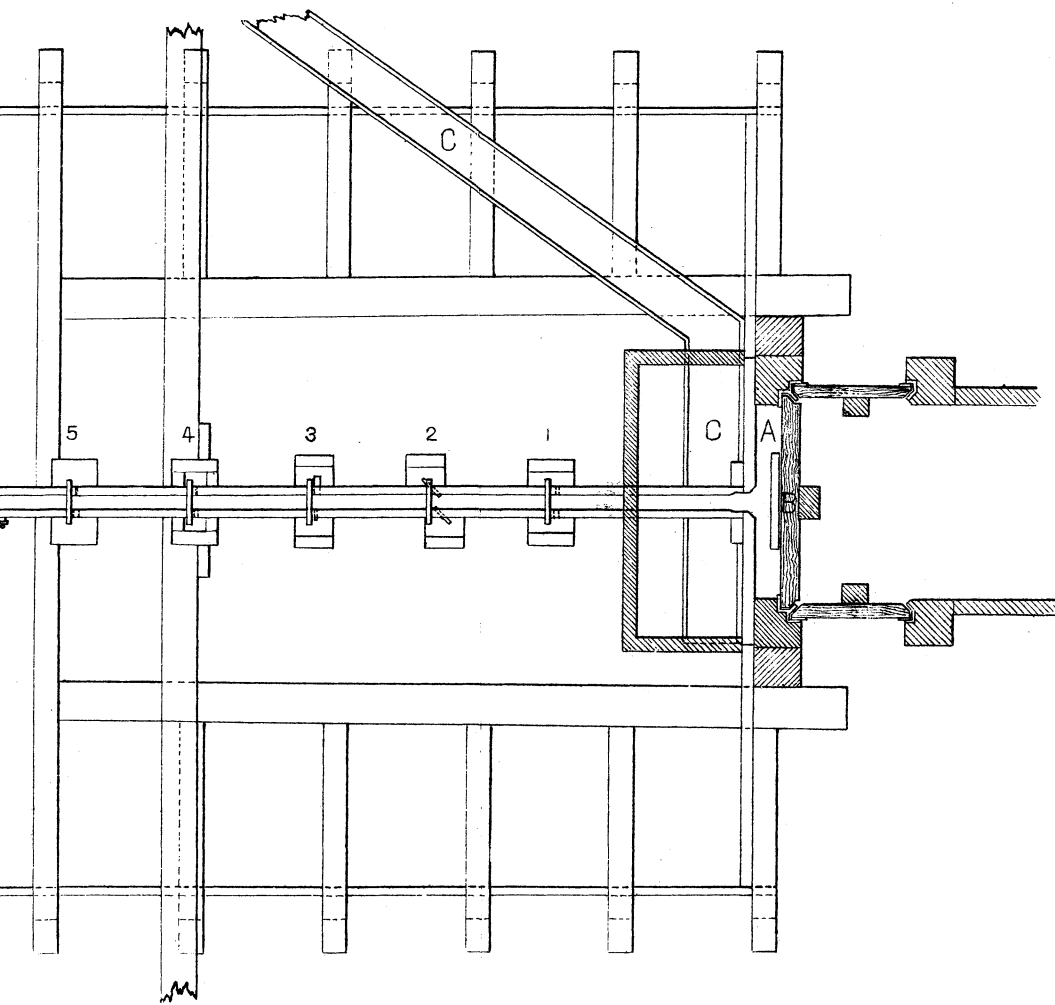
  

P L A T E I.									
	B A N D I V.			B A N D V I.			B A N D V I I.		
	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.	Indivdual Errors.	Accumulated Errors.	Indivdual Errors.
	+8	+8	+5	+5	+5	+5	+5	+5	+8
	-9	-9	-18	-18	-23	-17	-23	-17	-9
	-32	-32	-42	-42	-42	-23	-42	-23	-32
	-70	-70	-70	-70	-70	-38	-70	-38	-70
	-109	-109	-106	-106	-106	-36	-106	-36	-109
	-150	-150	-145	-145	-145	-41	-145	-41	-150
	-182	-182	-177	-177	-177	-32	-177	-32	-182
	-208	-208	-203	-203	-203	-26	-203	-26	-208
	-239	-239	-222	-222	-222	-21	-222	-21	-239
	-240	-240	-235	-235	-235	-11	-235	-11	-240
	-235	-235	-231	-231	-231	-5	-231	-5	-235
	-222	-222	-213	-213	-213	+13	-213	+13	-222
	-191	-191	-182	-182	-182	+31	-182	+31	-191
	-164	-164	-156	-156	-156	+27	-156	+27	-164
	-128	-128	-126	-126	-126	+30	-126	+30	-128
	-96	-96	-96	-96	-96	+33	-96	+33	-96
	-66	-66	-69	-69	-69	+29	-69	+29	-66
	-44	-44	-44	-44	-44	+22	-44	+22	-44
	-20	-20	-20	-20	-20	+24	-20	+24	-20
	-6	-6	-4	-4	-4	+14	-4	+14	-6

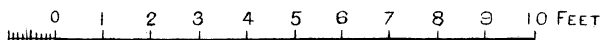
PLATE II.						PLATE III.						PLATE IV.			
BAND I.			BAND II.			BAND I.			BAND I.			BAND I.		BAND I.	
Individual Errors.	Accumulated Errors.		Individual Errors.	Accumulated Errors.		Individual Errors.	Accumulated Errors.		Individual Errors.	Accumulated Errors.		Individual Errors.	Accumulated Errors.	Individual Errors.	Accumulated Errors.
1 -17	-17	-6	-6	-6	+10	-5	-5	3	+3	+3	3	+15	+15	+15	+15
2 -1	-18	-10	-10	-16	+10	+6	+6	+9	+6	+6	+9	+1	+1	+1	+16
3 +15	-3	+2	+2	-14	-7	+5	+5	+17	+8	+8	+17	-2	-2	-2	+14
4 +1	-2	+4	+4	-10	+10	+7	+7	+14	+3	+3	+14	-10	-10	-10	+4
5 +6	-11	-6	-6	-21	+1	+2	+2	+17	+6	+6	+17	+7	+7	+7	+11
6 +6	-5	-11	-11	-27	+2	+12	+12	+23	+4	+4	+23	-3	-3	-3	+8
7 +1	-8	+1	+1	-34	-13	+2	+2	+27	+16	+16	+27	-7	-7	-7	+1
8 +3	-11	+4	+4	-33	-7	+15	+15	+46	+3	+3	+46	-6	-6	-6	+5
9 +1	-10	+4	+4	-29	+7	-2	-2	+32	+6	+6	+40	+5	+5	+5	+0
10 +5	-15	+4	+4	-32	-2	-6	-6	+26	-1	-1	+37	-1	-1	-1	+1
11 +1	-16	+0	+0	-32	-8	-6	-6	+20	-3	-3	+40	-6	-6	-6	+6
12 -1	-1	+21	+21	-32	+10	+2	+2	+22	-4	-4	+33	+10	+10	+10	+4
13 -4	-19	-21	-21	-53	-6	+9	+9	+24	-2	-2	+31	+16	+16	+16	+20
14 +5	-18	+3	+3	-50	-5	+9	+9	+33	+0	+0	+31	-5	-5	-5	+15
15 +5	-14	-2	-2	-52	-3	-14	-14	+19	+8	+8	+23	-12	-12	-12	+3
16 +15	-4	+13	+13	-39	-12	-5	-5	+14	+0	+0	+23	+2	+2	+2	+5
17 -2	-1	+23	+23	-16	+7	-9	-9	+5	-10	-10	+13	-10	-10	-10	+5
18 -2	-3	+2	+2	-18	+7	-9	-9	-9	-7	-7	+6	+13	+13	+13	+8
19 +7	-4	+5	+5	-13	+11	-5	-5	-9	-6	-6	+0	+5	+5	+5	+13
20 +1	-3	+4	+4	-9	+3	-5	-5	-12	+3	+3	+3	-12	-12	-12	+1
21 +1	-3	+1	+1	-8	+3	+0	+0	+8	+3	+3	+3	+3	+3	+3	+3
22 +1	-2	+6	+6	-14	-5	+1	+1	+5	+5	+5	+5	+3	+3	+3	+3
23 +1	-2	+1	+1	-13	+4	+1	+1	+1	+1	+1	+1	+3	+3	+3	+3
24 +6	-6	+5	+5	-8	+4	+0	+0	+1	+0	+0	+1	+3	+3	+3	+3
25 -4	-2	+2	+2	-6	-1	+1	+1	-6	-1	-1	-6	+3	+3	+3	+3







SCALE  $\frac{1}{48}$





# PLATE N<sup>o</sup>.3.

FIG. N<sup>o</sup>.1.

AVERAGE HEIGHTS AT STA'S. N<sup>o</sup>.4, N<sup>o</sup>.8, & N<sup>o</sup>.9.

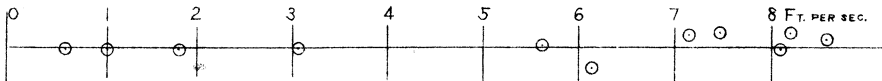


FIG. N<sup>o</sup>.2.

STA. N<sup>o</sup>.5, WEST SIDE,  $\square$  EAST SIDE,  $\circ$

STA. N<sup>o</sup>.9, WEST SIDE,  $+$

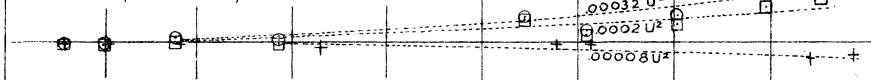


FIG. N<sup>o</sup>.3.

STA. N<sup>o</sup>.7, WEST SIDE,  $\square$  EAST SIDE,  $\circ$

STA. N<sup>o</sup>.10, WEST SIDE,  $+$  EAST SIDE,  $\Delta$

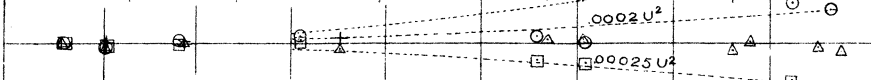
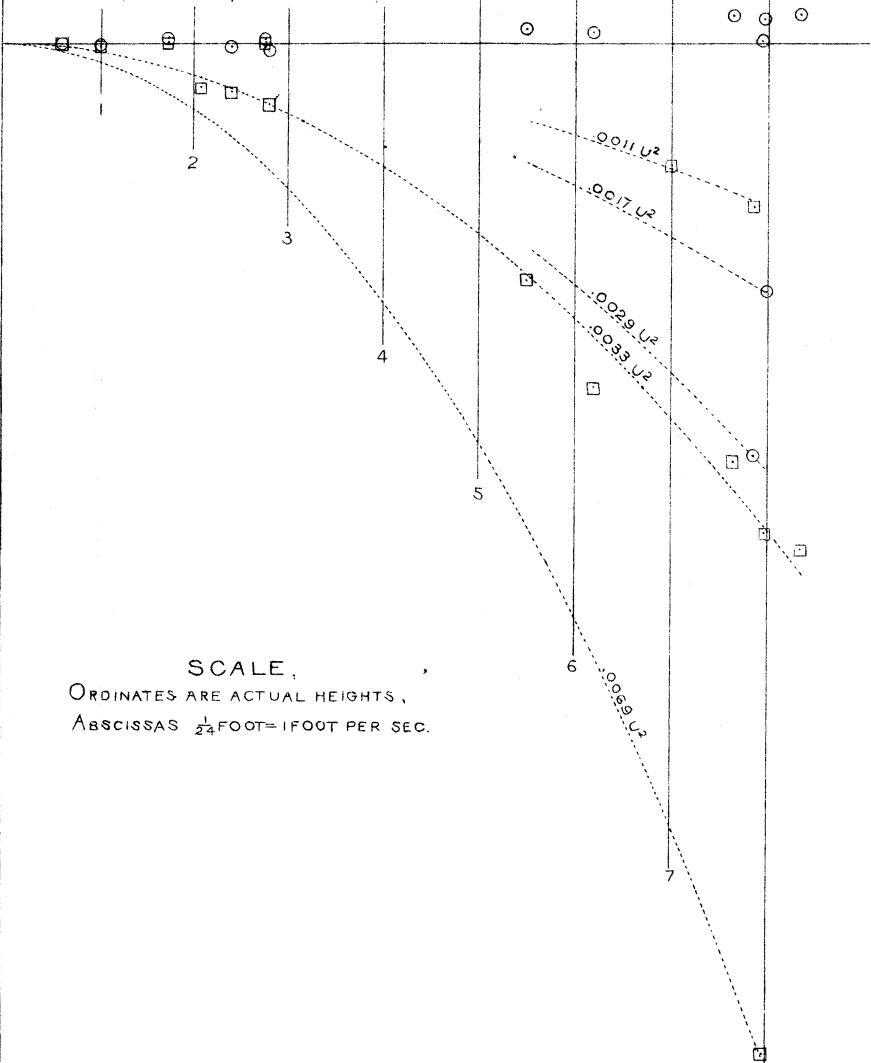


FIG. N<sup>o</sup>.4.

STA. N<sup>o</sup>.3, WEST SIDE,  $\square$  EAST SIDE,  $\circ$



SCALE,

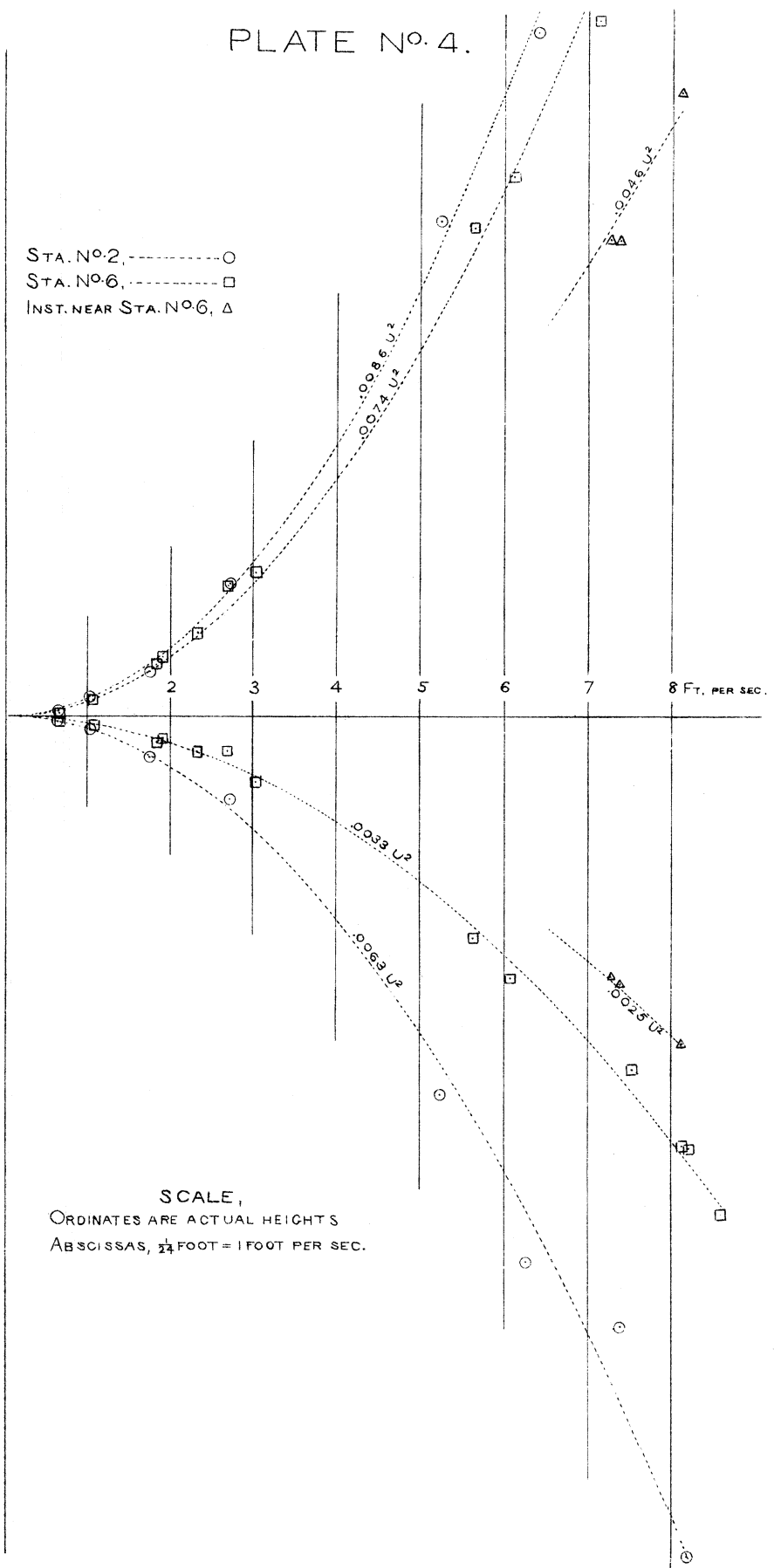
ORDINATES ARE ACTUAL HEIGHTS,

ABSCISSAS  $\frac{1}{24}$  FOOT = 1 FOOT PER SEC.

8 FT. PER SEC.

# PLATE N<sup>o</sup>. 4.

STA. N<sup>o</sup>. 2, ---○  
 STA. N<sup>o</sup>. 6, ---□  
 INST. NEAR STA. N<sup>o</sup>. 6, Δ



SCALE,  
 ORDINATES ARE ACTUAL HEIGHTS  
 ABSCISSAS,  $\frac{1}{24}$  FOOT = 1 FOOT PER SEC.